

Mathematical Modelling of Carbon Fee and Dividend (aka Climate Income)

Dave Waltham, May 3rd 2021

The [CCL-UK climate income calculator](#) assumes that household emissions increase with household income. A simple mathematical model which fits available data is ([Büchs & Schnepf, 2013](#))

$$E_0 = aI^b \quad (1)$$

where E_0 is household emissions (at the model start) and I is household disposable income. For the UK, $b \sim 0.6$. In the CCL-UK model, I determine a by assuming that a household of two adults, each earning the UK median income, has twice the average per capita emissions. Eqn (1) then gives $a=0.02$ (if income is in £/year).

In addition, emissions change with time, t , by a factor R so that emissions are

$$E(t) = R(t)E_0 \quad (2)$$

Household carbon-fees are then

$$F = PE \quad (3)$$

where F is the fee and P is the price. The dividend (i.e. climate income per adult) is found by assuming that an individual with average emissions will receive a dividend equal to their carbon-fees. Hence, the dividend per person is

$$D = PR\bar{E}_0 \quad (4)$$

where \bar{E}_0 is the average emissions per person at the beginning. However, this assumes that 100% of the revenues will be recycled (i.e. returned as a dividend). In practice, some funds will be needed to run the scheme and so I assume that D is reduced by a further recycling-rate factor, r , so that eqn (4) becomes

$$D = PR\bar{E}_0 r. \quad (5)$$

I use $r=0.95$ in the CCL-UK calculator.

Emission reductions are assumed to be driven by the carbon price (i.e. R decreases as P increases). Many choices could be made for the mathematical form of this but I have used the sigmoidal function

$$R = \begin{cases} \left[0.5 + 0.5 \cos\left(\frac{\pi P}{P_{max}}\right) \right] & P < P_{max} \\ 0 & P \geq P_{max} \end{cases} \quad (5)$$

where P_{max} is the price needed to eliminate emissions entirely. This sigmoidal form fits the [REMI model](#) of the US economy for $P_{max} \sim \$400/\text{tonne CO}_2$. The CCL-UK calculator uses a more conservative estimate that $P_{max} = £560/\text{tonne CO}_2$ so that it reproduces [Policy Exchange's](#) predictions for the UK.

Finally, the model needs a price policy and I assume a simple ramping up with time

$$P = P_{start} + \alpha t \quad (6)$$

where P_{start} is the initial price, α is the price increase each year and t is time since carbon pricing was introduced. More specifically, I have used $P_{start} = £10/\text{tonne}$ and $\alpha = £15/\text{tonne/year}$.